

SUPERSYMMETRIC $SO(10)$ GUT MODELS WITH YUKAWA UNIFICATION AND A POSITIVE μ TERM

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Abstract

Supersymmetric grand unified models based on $SO(10)$ gauge symmetry have many desirable features, including the unification of Yukawa couplings. Using D -term contributions to scalar masses arising from the breakdown of $SO(10)$, Yukawa coupling unification better than 15% can be achieved in models with a positive superpotential Higgs mass. The superparticle mass spectrum is highly constrained, and yields relatively light top squarks and charginos. Surprisingly, the pattern of GUT scale soft SUSY breaking masses are close to those found in the context of inverted hierarchy models. Our analysis supports the idea that the low energy MSSM parameter space is an approximate $SO(10)$ inspired fixed point.

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Supersymmetric grand unified theories (SUSYGUTS) based on the gauge group $SO(10)$ are especially attractive [1]. Not only do they unify the three forces of the Standard Model (SM), but they unify the matter content of each generation into a single 16 dimensional irreducible representation of $SO(10)$: $\hat{\psi}(\mathbf{16})$. The $\hat{\psi}(\mathbf{16})$ includes not only the matter superfields of the minimal supersymmetric standard model (MSSM), but also a gauge singlet superfield \hat{N}^c which includes a right handed neutrino. The gauge singlet superpotential mass term M_N can be of order M_{GUT} , and leads to sub-eV scale masses for left handed neutrinos (in accord with SuperK results on atmospheric neutrino oscillations), while the right handed neutrinos decouple via the well-known see-saw mechanism [2]. In the simplest models, the two Higgs doublet superfields of the MSSM reside in a single ten dimensional representation $\hat{\phi}(\mathbf{10})$. Then $SO(10)$ SUSYGUT models contain a superpotential interaction term

$$\hat{f} \ni f \hat{\psi} \hat{\psi} \phi + \dots \quad (1)$$

where f is the Yukawa coupling which leads to masses for quarks and leptons (at this stage we neglect intergenerational mixing effects). Thus, the Yukawa couplings of each generation are assumed to be unified above the GUT scale. More sophisticated treatments of the Yukawa matrices can allow for predictions of all SM fermion masses and mixing angles in terms of just a few parameters [3]. Here we will focus only on third generation Yukawa couplings, since they will be large, and can have a substantial impact on the spectrum of superpartners.

In much the same way that the three gauge couplings of the MSSM can be extrapolated from their weak scale values to their GUT scale values using renormalization group (RG) evolution, so too can the Yukawa couplings be evolved from the weak to the GUT scale to test models with Yukawa coupling unification. In the bottom-up approach used in ISAJET v7.54 [4], we begin with the weak scale values of m_b , m_t and m_τ in the \overline{DR} regularization scheme. We calculate the corresponding Yukawa couplings, and evolve both Yukawa and gauge couplings to higher energies using two-loop RG equations (RGEs) [5]. Once M_{GUT} is determined (by the point at which the $SU(2)$ and $U(1)$ gauge couplings meet), we evolve gauge couplings, Yukawa couplings and soft SUSY breaking (SSB) mass terms to the weak scale M_{weak} , where electroweak symmetry is broken radiatively (REWSB), and where the entire SUSY particle mass spectrum can be calculated. At this stage, the Yukawa couplings can be updated to include SM and MSSM loop corrections [6,9], and the RGE process is iterated until a convergent spectrum of SUSY particle masses is obtained. In this way, the GUT scale values of the Yukawa couplings depend on the SUSY particle mass spectrum.

In previous reports [10–12], unification of Yukawa couplings was investigated within the context of $SO(10)$ SUSYGUT models. It is well known that in the mSUGRA model, with universal SSB masses at the GUT scale, a high degree of Yukawa coupling unification only occurs for negative values of the superpotential Higgs mass term μ , and for values of the ratio of Higgs field vevs $\tan \beta \equiv \frac{v_u}{v_d} \sim 50$ [9,11]. For such high values of $\tan \beta$, the scalar potential no longer has the appropriate form at the weak scale to accommodate REWSB. Generally, the SSB down Higgs mass squared $m_{H_d}^2$ gets driven to negative values before the up Higgs mass squared $m_{H_u}^2$. However, even if scalar masses are universal above the GUT scale, they will receive D -term mass contributions at the GUT scale arising from the breakdown of $SO(10)$ gauge symmetry. Thus, scalar masses are shifted by an amount

$$m_Q^2 = m_E^2 = m_U^2 = m_{16}^2 + M_D^2$$

$$\begin{aligned}
m_D^2 &= m_L^2 = m_{16}^2 - 3M_D^2 \\
m_N^2 &= m_{16}^2 + 5M_D^2 \\
m_{H_{u,d}}^2 &= m_{10}^2 \mp 2M_D^2
\end{aligned}$$

where M_D^2 parametrizes our ignorance of the exact breakdown mechanism for $SO(10)$, and can have either positive or negative values of order M_{weak}^2 . Thus, the model is characterized by the following free parameters:

$$m_{16}, m_{10}, M_D^2, m_{1/2}, A_0, \tan\beta \text{ and } \text{sign}(\mu). \quad (2)$$

The value of $\tan\beta$ will be restricted by the requirement of Yukawa coupling unification to be close to ~ 50 . In this model, for positive values of M_D^2 , the GUT scale values of $m_{H_u}^2$ and $m_{H_d}^2$ are split, and $m_{H_u}^2$ gets a head start on running towards negative values. For a sufficiently large value of M_D^2 , REWSB can be recovered, and viable supersymmetric models with a high degree of third generation Yukawa coupling can be generated [11].

In previous works, model parameter space was mapped out under the restriction of GUT scale Yukawa unification to 5% [11], and implications of the model for cosmological neutralino relic density, direct detection of dark matter, radiative decays $b \rightarrow s\gamma$ and collider searches were determined [12]. A favorable relic density was obtained over much of model parameter space owing to s -channel neutralino pair annihilation via the very wide A and H Higgs poles at large $\tan\beta$ [13]. However, the decay width for $b \rightarrow s\gamma$ was found to be very large and generally in disagreement with experimental limits unless one entered the decoupling regime, where model parameters began becoming unnatural. The large $b \rightarrow s\gamma$ branching fraction at large $\tan\beta$ and $\mu < 0$ is well known [14]. However, for $\mu > 0$, the $b \rightarrow s\gamma$ branching ratio can be in accord with experimental limits at large $\tan\beta$.

In addition, the recent measurement by the E821 experiment [15] of the muon anomalous magnetic moment $a_\mu = (g - 2)/2$ was found to deviate from SM predictions [16] by 2.6σ . If the deviation is interpreted in terms of supersymmetric models, then it disfavors models with $\mu < 0$, assuming positive SSB gaugino masses [17]. For these reasons, it seemed prudent to re-examine Yukawa unification for positive values of μ , relaxing the ad-hoc 5% criteria used in Refs. [11,12].

Our procedure is as follows. We generate random samples of model parameters

$$\begin{aligned}
0 &< m_{16} < 2000 \text{ GeV}, \\
0 &< m_{10} < 2500 \text{ GeV}, \\
0 &< m_{1/2} < 1000 \text{ GeV}, \\
-1000^2 &< M_D^2 < +1000^2 \text{ GeV}^2, \\
45 &< \tan\beta < 55, \\
-5000 &< A_0 < 5000 \text{ GeV and} \\
\mu &> 0.
\end{aligned}$$

We then calculate the non-universal scalar masses according to formulae given above, and enter the parameters into the computer program ISASUGRA. ISASUGRA is a part of the ISAJET package [4] which calculates an iterative solution to the 26 coupled RGEs of the MSSM. We required unification of third generation Yukawa couplings at the GUT scale to

15%. Our requirement of 15% unification is determined by defining the variables $r_{b\tau}$, r_{tb} and $r_{t\tau}$, where for instance $r_{b\tau} = \max(f_b/f_\tau, f_\tau/f_b)$. We then require $R = \max(r_{b\tau}, r_{tb}, r_{t\tau}) < 1.15$. We were able to find many solutions fulfilling the above criteria, with the best overall unification achieved for $R = 1.08$, or Yukawa coupling unification to 8%. The top and tau Yukawa couplings could frequently unify to very high precision, while $f_b/f_t \sim 0.87 - 0.95$, and $f_b/f_\tau \sim 0.87 - 0.92$. The Yukawa couplings at the GUT scale, $f_i(M_{GUT})$, differ from the unified Yukawa, f_{GUT} , due to threshold corrections, $f_i(M_{GUT}) = f_{GUT}(1 + \epsilon_i)$. Threshold effects could explain our 8% deviation from perfect Yukawa unification [6–8]. On the other hand these thresholds are GUT model dependent. Thus, $SO(10)$ SUSYGUT model builders should look towards models which yield GUT scale threshold corrections of these magnitudes.

The parameter space regions with Yukawa coupling unification to 15% are shown in Fig. 1. The values of $\tan\beta$ generated were within the narrow range of $\tan\beta \sim 46 - 48$, typically somewhat lower than results assuming $\mu < 0$. In frame *a*), we show models in the m_{16} vs. $m_{1/2}$ plane. The values of m_{16} and m_{10} typically lie beyond 1 TeV and cluster about the line $m_{10} = \sqrt{2}m_{16}$.

In frame *b*), we show solutions in the m_{16} vs. M_D plane. In this case, we find M_D restricted to values of 0.1 – 0.4 TeV. No solutions were obtained with $M_D \simeq 0$, which would bring us back to the mSUGRA model. Thus, the $SO(10)$ D -terms are crucial to Yukawa coupling unification. We find unified solutions only for $m_{H_d} > m_{H_u}$; this requirement puts a positive lower bound on M_D^2 .

In frame *c*), we show the m_{16} vs. A_0 parameter plane. Surprisingly, Yukawa unified solutions are found only for large negative values of the A_0 parameter, and cluster about the line $A_0 = -2m_{16}$. Finally, in frame *d*), we show the m_{16} vs. $m_{1/2}$ plane, and find model solutions occurring for $m_{1/2} \sim 0.1 - 0.9$ TeV, with m_{16} always greater than $m_{1/2}$.

It is particularly intriguing that model solutions with Yukawa coupling unification cluster about model parameters with

$$A_0^2 = 2m_{10}^2 = 4m_{16}^2. \quad (3)$$

These particular boundary conditions were found by Bagger *et al.* from a rather different approach [18], by looking analytically for fixed point behavior in third generation SSB scalar masses which would give rise to SUSY models with a radiatively driven inverted scalar mass hierarchy (RIMH). In these models, one may begin with GUT scale scalar masses beyond the TeV scale. RG evolution drives third generation and Higgs scalar masses towards zero, while scalars of the first two generations remain beyond the TeV range. In this way, multi-TeV first and second generation scalar masses act to suppress SUSY flavor and CP violating processes, while still fulfilling conditions of naturalness, which mainly apply to the sub-TeV third generation and Higgs scalar SSB masses. The RIMH mechanism is viable for $SO(10)$ based models with Yukawa coupling unification upon implementation of the above specific set of scalar mass boundary conditions [19,20]. Our results here are obtained using a bottom-up approach, and indicate that for $\mu > 0$, a high degree of Yukawa coupling unification can *only* be obtained using approximately the boundary conditions Eq. 3. The values of the Yukawa couplings obtained at the GUT scale are $f_t : 0.49 - 0.52$, $f_b : 0.43 - 0.48$ and $f_\tau : 0.49 - 0.52$. Their magnitudes are only sufficient to generate a small scalar IMH [19,20]. For Yukawa unified solutions with $\mu < 0$, we find no real correlation of solutions with Eq. 3.

In Fig. 2, we show values of selected weak scale sparticle masses generated from the

Yukawa unified model. In frame *a*), we show the m_A *vs.* m_h plane of Higgs masses. The light scalar h has a mass clustering about the region $m_h \sim 115 - 130$ GeV, while m_A ranges between $100 - 500$ GeV, and is generally lower than m_A values generated in models with lower values of $\tan\beta$. Both the h and A (and also the heavy Higgs H) may be accessible to Higgs searches at the Fermilab Tevatron [21], and the low values of m_A may give rise to measureable rates for $B \rightarrow \mu^+\mu^-$ decay [22].

In frame *b*), we show the $m_{\tilde{t}_1}$ *vs.* $m_{\tilde{b}_1}$ plane. We always find $m_{\tilde{b}_1} > m_{\tilde{t}_1}$, in contrast to Yukawa unified models with $\mu < 0$. The value of $m_{\tilde{t}_1}$ ranges between $200 - 600$ GeV for $m_{\tilde{b}_1} < 1$ TeV.

Frame *c*) shows the $m_{\tilde{\chi}_1^\pm}$ *vs.* $m_{\tilde{\tau}_1}$ plane. We note that $m_{\tilde{\tau}_1} \gtrsim m_{\tilde{\chi}_1^\pm}$, while $m_{\tilde{\chi}_1^\pm} \lesssim 500$ GeV, and is likely accessible to a linear e^+e^- collider operating with $\sqrt{s} \sim 1$ TeV [23,24].

Finally, in frame *d*), we show the $m_{\tilde{e}_R}$ *vs.* $m_{\tilde{u}_R}$ plane. In this case, $m_{\tilde{e}_R}$ and $m_{\tilde{u}_R}$ typically lie beyond 1 TeV. Such high mass values for first and second generation scalars can act to suppress many CP violating processes via a decoupling solution; they are generally not sufficiently heavy to suitably suppress the most dangerous flavor violating processes, such as $K - \bar{K}$ mixing [25]. A sample sparticle mass spectrum is shown in Table 1 for a Yukawa unified model with $\mu > 0$.

The cosmological relic density of neutralinos has been calculated in Ref. [12] for Yukawa unified models with $\mu < 0$. Little should change by switching to $\mu > 0$: the pseudoscalar A and heavy scalar Higgs H will still have large widths of order $20 - 50$ GeV due to the large b and τ Yukawa couplings, and will be light enough that $\tilde{\chi}_1^0\tilde{\chi}_1^0$ annihilation can take place efficiently through s -channel annihilation. In addition, the rates for direct detection of relic neutralinos will remain large, as in the $\mu < 0$ case [12]. The rate for $b \rightarrow s\gamma$ can be substantially different for $\mu > 0$ compared to the $\mu < 0$ result, and regions of parameter space certainly exist where this decay rate falls within experimental limits. Explicit results for the RIMH model with $\mu > 0$ have been shown in Ref. [20]. Finally, the value of a_μ has been calculated in Ref. [17] for Yukawa unified models with $\mu < 0$ and for RIMH models with $\mu > 0$. Regions of model space with an acceptable a_μ certainly exist for the $\mu > 0$ case. Further results along these lines will be presented in a forthcoming publication.

Finally, we note that Yukawa unified models with $\mu > 0$ have also been recently reported by Blazek *et al.* [26]. These authors use a top-down approach and adopt independent values for m_{H_u} and m_{H_d} rather than D -term splitting amongst scalar masses. We verify that in this case also Yukawa unified solutions can be obtained, although in our approach these generally occur at the 12-25% level. The solutions typically have $m_{H_u} \sim \sqrt{2}m_{16}$, with $m_{H_u} < m_{H_d}$, and $A_0 \simeq -2m_{16}$.

It is well known that the three standard model gauge couplings approximately unify when extrapolated to the scale $M_{GUT} \simeq 2 \times 10^{16}$ GeV. This may be regarded as a coincidence, or as evidence for SUSYGUTs. Similarly, as a consequence of Yukawa unification, the clustering of the SSB parameters about an approximate $SO(10)$ inspired fixed point, if taken seriously, can be considered as evidence for a supersymmetric $SO(10)$ grand unified model.

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REFERENCES

- [1] H. Georgi, in *Proceedings of the American Institute of Physics*, edited by C. Carlson (1974); H. Fritzsch and P. Minkowski, *Ann. Phys.* **93**, 193 (1975); M. Gell-Mann, P. Ramond and R. Slansky, *Rev. Mod. Phys.* **50**, 721 (1978). For a recent review, see R. Mohapatra, hep-ph/9911272 (1999).
- [2] M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity, Proceedings of the Workshop*, Stony Brook, NY 1979 (North-Holland, Amsterdam); T. Yanagida, KEK Report No. 79-18, 1979; R. Mohapatra and G. Senjanovic, *Phys. Rev. Lett.* **44**, 912 (1980).
- [3] Some references include G. Anderson, S. Raby, S. Dimopoulos, L. Hall and G. Starkman, *Phys. Rev. D***49**, 3660 (1994); M. Carena, S. Dimopoulos, C. Wagner and S. Raby, *Phys. Rev. D***52**, 4133 (1995); K. Babu, J. Pati and F. Wilczek, *Nucl. Phys.* **B566**, 33 (2000); C. Albright and S. Barr, *Phys. Rev. D***58**, 013002 (1998).
- [4] F. Paige, S. Protopopescu, H. Baer and X. Tata, hep-ph/0001086 (2000).
- [5] S. Martin and M. Vaughn, *Phys. Rev. D***50**, 2282 (1994).
- [6] L. J. Hall, R. Rattazzi and U. Sarid, *Phys. Rev. D***50**, 7048 (1994).
- [7] P. Langacker and N. Polonsky, *Phys. Rev. D* **47**, 4028 (1993)
- [8] J. Bagger, K. Matchev and D. Pierce, *Phys. Lett. B* **348**, 443 (1995)
- [9] D. Pierce, J. Bagger, K. Matchev and R. Zhang, *Nucl. Phys.* **B491**, 3 (1997).
- [10] M. Carena, M. Olechowski, S. Pokorski and C. Wagner, *Nucl. Phys.* **B426**, 269 (1994); B. Ananthanarayan, Q. Shafi and X. Wang, *Phys. Rev. D***50**, 5980 (1994); R. Rattazzi and U. Sarid, *Phys. Rev. D***53**, 1553 (1996).
- [11] H. Baer, M. Diaz, J. Ferrandis and X. Tata, *Phys. Rev. D***61**, 111701 (2000).
- [12] H. Baer, M. Brhlik, M. Diaz, J. Ferrandis, P. Mercadante, P. Quintana and X. Tata, *Phys. Rev. D***63**, 015007 (2001).
- [13] M. Drees and M. Nojiri, *Phys. Rev. D***47**, 376 (1993); H. Baer and M. Brhlik, *Phys. Rev. D***57**, 567 (1998).
- [14] H. Baer, M. Brhlik, D. Castano and X. Tata, *Phys. Rev. D***58**, 015007 (1998).
- [15] H. N. Brown *et al.* (Muon $g - 2$ Collaboration), *Phys. Rev. Lett.* **86**, 2227 (2001).
- [16] A. Czarnecki and W. Marciano, *Phys. Rev. D***64**, 013014 (2001).
- [17] See *e.g.* H. Baer, C. Balazs, J. Ferrandis and X. Tata, *Phys. Rev. D***64**, 035004 (2001) and references therein.
- [18] J. Bagger, J. Feng, N. Polonsky and R. Zhang, *Phys. Lett.* **B473**, 264 (2000).
- [19] H. Baer, P. Mercadante and X. Tata, *Phys. Lett.* **B475**, 289 (2000).
- [20] H. Baer, C. Balázs, M. Brhlik, P. Mercadante, X. Tata and Y. Wang, *Phys. Rev. D***64**, 015002 (2001).
- [21] M. Carena *et al.*, hep-ph/0010338 (2000).
- [22] K. Babu and C. Kolda, *Phys. Rev. Lett.* **84**, 228 (2000).
- [23] Physics at TESLA, R.D. Heuer *et al.*, DESY-01-011C (2001).
- [24] T. Abe *et al.* (American Linear Collider Working Group), hep-ex/0106057 (2001).
- [25] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, *Nucl. Phys.* **B477**, 321 (1996).
- [26] T. Blazek, R. Dermisek and S. Raby, hep-ph/0107097 (2001).

TABLES

TABLE I. Weak scale sparticle masses and parameters (GeV) for an $SO(10)$ case study.

parameter	value
m_{16}	1434.7
m_{10}	1915.9
M_D	282.6
$m_{1/2}$	334.1
A_0	-2722.7
$\tan \beta$	47.0
$f_t(M_{GUT})$	0.505
$f_b(M_{GUT})$	0.467
$f_\tau(M_{GUT})$	0.511
$m_{\tilde{g}}$	847.5
$m_{\tilde{u}_L}$	1580.8
$m_{\tilde{d}_R}$	1472.4
$m_{\tilde{t}_1}$	296.6
$m_{\tilde{b}_1}$	515.0
$m_{\tilde{\ell}_L}$	1357.5
$m_{\tilde{\ell}_R}$	1471.1
$m_{\tilde{\nu}_e}$	1355.1
$m_{\tilde{\tau}_1}$	691.7
$m_{\tilde{\nu}_\tau}$	1003.2
$m_{\tilde{\chi}_1^\pm}$	113.3
$m_{\tilde{\chi}_2^0}$	137.1
$m_{\tilde{\chi}_1^0}$	91.7
m_h	124.1
m_A	251.9
m_{H^\pm}	271.3
μ	125.7
a_μ	13.3×10^{-10}

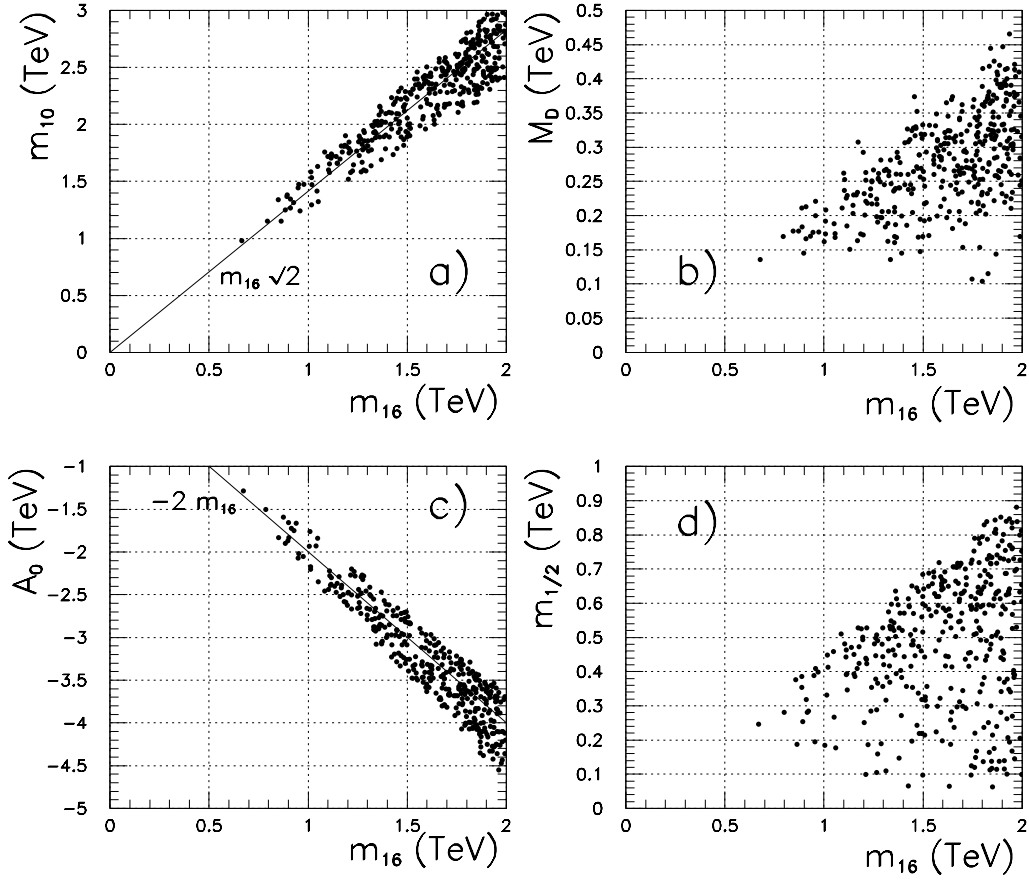


FIG. 1. Plots of regions of parameter space where valid solutions to minimal SUSY $SO(10)$ are obtained, consistent with Yukawa coupling unification to 15% with $\mu > 0$ and REWSB.

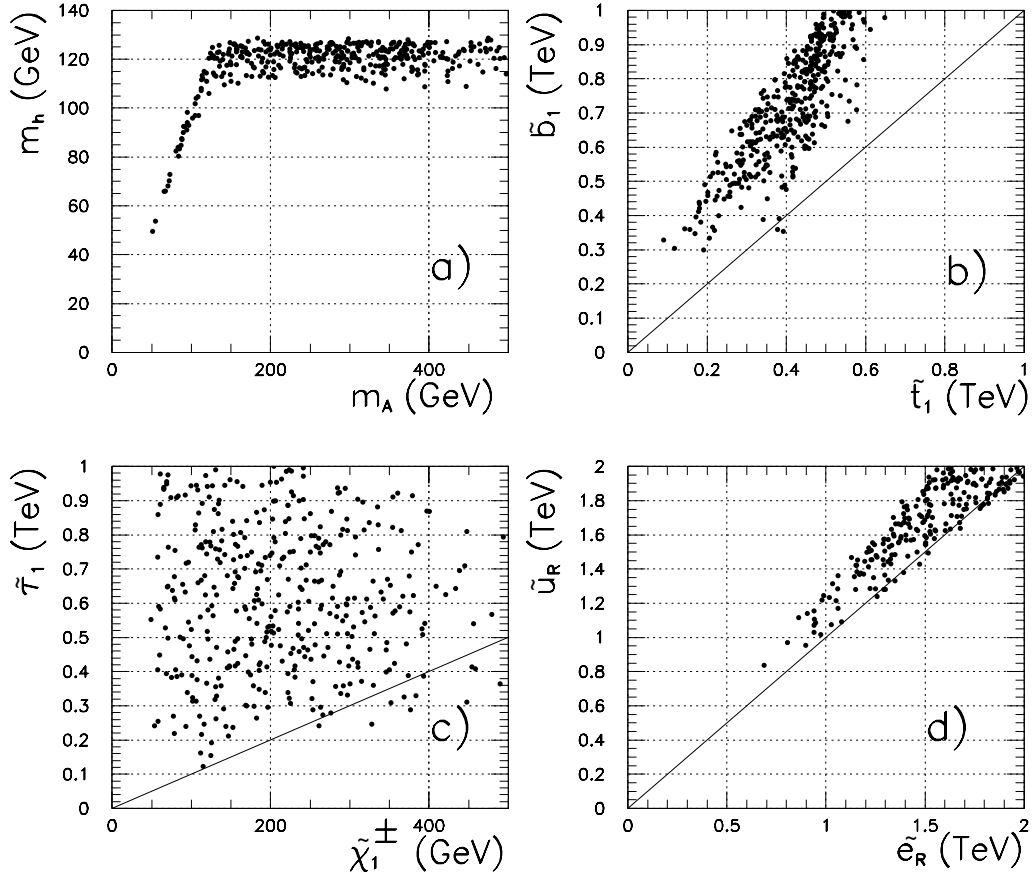


FIG. 2. The range of selected particle masses that are generated in minimal SUSY $SO(10)$ models with Yukawa coupling unification to 15%, $\mu > 0$ and REWSB.